Content Descriptions

Based on the Georgia Performance Standards

Mathematics II:
Geometry/Algebra II/Statistics

GEORGIA
DEPARTMENT OF
EDUCATION

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Introduction
The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated in 2001 to be tested. After the Georgia Performance Standards (GPS) were adopted by the State Board of Education in July 2004, Georgia educators began to develop the Mathematics II EOCT based on these standards.

Program Purpose
The EOCT are designed to improve student achievement by assessing student performance on the standards specific to each course tested. Student performance on each EOCT is provided to schools for diagnostic and remedial use. These results are used to help make instruction more effective and to ensure that all Georgia students have access to a rigorous curriculum that meets high academic standards. These results are also used for student accountability and to gauge the quality of education in the state. The EOCT are the final exams for each EOCT course. The final grade in each course is calculated by weighing the course grade 85% and the EOCT score 15%. The student must have a final grade of at least 70 to pass the course and to earn credit toward graduation.

EOCT Content Descriptions
The EOCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the EOCT. Only the knowledge, concepts, and skills addressed in the GPS are assessed on the EOCT. Committees of Georgia educators reviewed the curriculum and provided guidance for the assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, paper-and-pencil assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the EOCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the EOCT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS are located at http://www.georgiastandards.org.

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Mathematics II Domains

To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics II:

- Algebra (including Number and Operations)
  Investigate piecewise, exponential, and quadratic functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations; solve equations and inequalities related to these functions; explore the inverses of functions; use the complex number system

- Geometry
  Understand and apply properties of right triangles and right-triangle trigonometry; understand and apply properties of circles and spheres, and use them in determining related measures

- Data Analysis and Probability
  Demonstrate understanding of data analysis by posing questions to be answered by collecting data; organize, represent, investigate, interpret, and make inferences from data; compare data for two different samples and/or populations using measures of central tendency and measures of spread, including standard deviation; use linear and quadratic regressions to analyze data and to make inferences

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Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the process standards are integrated in items across the three content domains.

Overview of the Process Standards
- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and to other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS Standards

MM2P1 through MM2P5 within content from MM2N1 through MM2D2

Associated Skills
- Building new mathematical knowledge through problem solving
- Solving problems that arise in mathematics and in other contexts
- Applying and adapting a variety of appropriate strategies to solve problems
- Monitoring and reflecting on the process of mathematical problem solving
- Recognizing reasoning and proof as fundamental aspects of mathematics
- Making and investigating mathematical conjectures
- Developing and evaluating mathematical arguments and proofs
- Selecting and using various types of reasoning and methods of proof
- Organizing and consolidating mathematical thinking through communication
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others
- Analyzing and evaluating mathematical thinking and strategies of others
- Using the language of mathematics to express mathematical ideas precisely
- Recognizing and using connections among mathematical ideas
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognizing and applying mathematics in contexts outside of mathematics
- Creating and using representations to organize, record, and communicate mathematical ideas

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- Selecting, applying, and translating among mathematical representations to solve problems
- Using representations to model and interpret physical, social, and mathematical phenomena

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Mathematics II
Domain: Algebra

Overview of the Domain

- Students will represent and operate with complex numbers.
- Students will investigate step and piecewise functions, including greatest integer and absolute value functions.
- Students will explore exponential functions.
- Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.
- Students will solve quadratic equations and inequalities in one variable.
- Students will explore the inverses of functions.

Associated GPS Standards

MM2N1   MM2A1   MM2A2   MM2A3   MM2A4   MM2A5

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- write square roots of negative numbers in imaginary form; write complex numbers in the form $a + bi$ in the context of solving quadratic equations
- add, subtract, multiply, and divide complex numbers
    - apply the associative, distributive, and commutative properties
    - identify and find conjugates of complex numbers
- simplify expressions involving complex numbers
    - understand that $i = \sqrt{-1}$ and $i^2 = -1$
- write and interpret absolute value functions as piecewise functions
    - recognize and write appropriate function rules for different parts of the domain
- investigate and explain characteristics of a variety of piecewise-defined functions, such as absolute value and greatest integer functions; relate these characteristics to a real-life situation modeled by such a function
    - translate fluently between graphical, algebraic, and numeric representations
    - identify the domain and range
    - find the vertex and axis of symmetry
    - identify the zeroes
    - find the x- and y-intercepts
    - identify points of discontinuity

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- – identify intervals where the value of a function is constant, increasing, or decreasing
- – investigate rates of change for specified intervals

- solve absolute value equations and inequalities
  - – use algebraic and analytical methods
  - – determine solutions using graphs and/or number lines
- extend properties of exponents to include all integer exponents and use expressions with integer exponents to model real-world functional relationships
  - – apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to simplify and/or evaluate expressions
  - – understand that for any real number a, $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$ and apply these properties
- investigate and explain characteristics of exponential functions; use these characteristics to model and solve real-world problems
  - – identify domain and range
  - – identify zeroes
  - – find x- and y-intercepts
  - – recognize and/or determine intervals where the value of a function is increasing or decreasing
  - – find maximum and minimum values
  - – investigate rates of change over intervals
  - – recognize and explain behavior at extremes
- graph exponential functions as transformations of $f(x) = a^x$
  - – recognize and use transformations of $f(x) = a^x$
  - – use tables of values
- solve simple exponential equations and inequalities
  - – by using algebraic and analytical methods
  - – by creating and interpreting graphs
- understand basic exponential functions as models of real phenomena and use exponential functions to solve problems
  - – apply simple and compound interest formulas
  - – recognize exponential growth and decay functions in problem situations and in numerical, graphical, and algebraic representations
  - – use exponential growth and decay models to solve problems; understand how the parameters of an exponential function relate to a situation modeled by that function
- understand and recognize geometric sequences as exponential functions whose domains are the sequence of natural (counting) numbers
  - – interpret the constant ratio in a geometric sequence as the base of the associated exponential function

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- – recognize and use concepts such as the common ratio and powers of the common ratio to solve real-world problems involving exponential growth and decay
- graph quadratic functions as transformations of the function $f(x) = x^2$
  - – identify vertical and horizontal stretches and compressions, and vertical and horizontal translations
  - – explore reflections across the x- and y-axes
- convert between standard $y = \left(ax^2 + bx + c\right)$ and vertex $y = \left(a\left(x - h\right)^2 + k\right)$ forms of a quadratic function using the roots of the quadratic and the symmetry properties of the parabola; use the vertex form to locate and graph a quadratic function, e.g., when using a quadratic function to model a data relationship; translate from vertex form back to standard form to identify the parameters a, b, and c
- investigate and explain characteristics of quadratic functions; use these characteristics to model and solve real-world problems
  - – identify domain and range
  - – identify the vertex and axis of symmetry
  - – find all zeroes
  - – find the x- and y-intercepts
  - – locate extrema using ordered pairs and be able to identify maximum and minimum values
  - – determine intervals of increase and decrease
  - – investigate rates of change for specific intervals
- explore arithmetic series and various ways of computing their sums
  - – calculate the sum of a series by representing it as a function, using patterns, or using a formula, e.g., $S_n = \dfrac{n}{2}(a_1 + a_2)$
  - – evaluate the sum of a finite series expressed in summation notation
- recognize sequences of partial sums of arithmetic series as examples of quadratic functions
  - – calculate the partial sum of a finite series by representing it as a function, using patterns, or using a formula, e.g., $S_n = \dfrac{n}{2}[2a_1 + (n - 1)d]$ or
  $S_n = \dfrac{n}{2}(a_1 + a_n)$
- solve quadratic equations graphically; use graphs to estimate solutions to quadratic functions and identify the number of real solutions and intervals on which the solutions occur
  - – interpret the x-intercepts of the function in the context of a real-life situation that is modeled by the function
- find real and complex solutions of quadratic equations analytically; be familiar with multiple methods and recognize when a certain method is most appropriate

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- – use factoring methods and the zero-product property
- – apply the quadratic formula
- – explore both exact and approximate solutions; recognize when each type of solution is appropriate and why
- – recognize how the solutions of quadratic equations apply to a real-world situation modeled by the quadratic function, e.g., when one or both roots are meaningless in context
- analyze the nature of roots of quadratic equations
  - – use a graph, a table of values, and/or the discriminant, $b^2 - 4ac$, to determine the number of real solutions; relate the value of the discriminant to the behavior of the graph and/or the context
- solve quadratic inequalities and describe the solutions using linear inequalities
  - – determine the solutions using one-variable inequalities obtained from factoring and apply the zero-product property; relate solutions to graphs on a number line; use the number line to investigate and identify solution intervals
  - – understand and apply conventions related to graphical representations of inequalities, e.g., shading and dotted lines vs. solid lines; use various strategies to test which part of a graph should be shaded and identify correct shading
- understand the relationship between a function and its inverse
  - – read, interpret, and use function and inverse function notation
  - – recognize and find the inverse of a function or relation using a variety of methods: interchanging the first and second coordinates of each ordered pair; graphing the function and its reflection across the line $y = x$; using analytical (algebraic) techniques; using composition of functions, i.e., exploring the identity function, $I(x) = x$, and how it is related to a function and its inverse; and/or determining that two functions, f and g, are inverses by recognizing or demonstrating that $(f \circ g)(x) = (g \circ f)(x) = I(x) = x$
  - – understand that functions with inverses that are also functions are considered to be one-to-one; understand how one-to-oneness relates to a real-world functional relationship
  - – recognize that the domain of the inverse is the range of the original relation and vice versa
  - – understand how and why domain restrictions come into play with inverse functions and relate them to the behavior of the original function
  - – understand and apply methods to characterize a relation as a function, including inspection, tables of values, and graphical methods such as the vertical line test

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Mathematics II
Domain: Geometry

Overview of the Domain
- Students will identify and use special right triangles.
- Students will define and apply sine, cosine, and tangent ratios to right triangles.
- Students will understand and apply the properties of circles and their associated segments and angles.
- Students will find and compare the measures of spheres.

Associated GPS Standards

MM2G1   MM2G2   MM2G3   MM2G4

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to
- determine the lengths of sides of 30°-60°-90° triangles
  - use the fact that the length of the hypotenuse is twice the length of the shorter leg and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg to determine the lengths of all three sides given any one of the three sides
  - solve problems that involve application of these side length relationships
- determine the lengths of sides of 45°-45°-90° triangles
  - use the fact that the length of the hypotenuse is $\sqrt{2}$ times the length of each leg to determine the lengths of all sides of a triangle given the length of any one of the three sides
  - solve problems that involve application of these side length relationships
- understand and apply the basic trigonometric ratios for right triangles
  - apply trigonometric ratios to find unknown measures of sides; solve real-life problems involving trigonometric ratios of right triangles; solve for the length of any side of a right triangle given sufficient information; work from a model presented graphically or verbally, including situations involving angles of elevation and/or depression
- understand and use properties of and relationships among radii, chords, tangents, and secants of circles as an application of triangle similarity, e.g.,
  - relationship of tangent to radius to point of tangency
  - congruence of tangents from a given point outside the circle
  - products of lengths of segments created by intersecting chords
  - products of lengths of segments created by a secant and a tangent from a given point outside the circle

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- understand and use properties of and relationships among angles related to circles, such as central, inscribed, and related angles, e.g.,
  - relationship between arc measures and angle measures
  - relationship between measures of central angles and inscribed angles
  - relationship of angles created by a chord with a common endpoint on the circle and the line that is tangent at that point
- use the properties of circles to solve problems involving the length of an arc and the area of a sector
- justify measurements and relationships in circles using geometric and algebraic properties
- understand, use and apply the surface area and volume of a sphere
  - calculate surface area and volume of a sphere
  - find the radius, diameter, and/or circumference of a sphere given the volume or a relationship from which the volume can be determined
  - determine the effect on surface area and volume when changing the radius or diameter of a sphere or vice versa

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Mathematics II
Domain: Data Analysis and Probability

Overview of the Domain
- Students will use sample data to make informal inferences using population means and standard deviations.
- Students will determine an algebraic (limited to linear or quadratic) model to quantify the association between two quantitative variables.

Associated GPS Standards

MM2D1   MM2D2

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to
- recognize an appropriate question given a research topic and populations of interest
- calculate the mean and standard deviation for a population or a sample; understand when the sample or population standard deviation formula is appropriate
- use means and standard deviations to compare data sets
  - understand and apply various strategies for estimating means and standard deviations for comparison purposes
  - understand various representations of data, including tables, graphs, line plots, stem-and-leaf plots, histograms, and box-and-whisker plots; know which information can be directly determined and which can only be estimated from a given representation
  - understand the role of n in comparing standard deviations of data sets, including recognizing when n is unknown
- compare the means and standard deviations of random samples drawn from a population with corresponding population parameters
  - understand that the distribution of the sample means has less variability than the population distribution
  - understand and apply the relationship between the number of samples, the sample variance, and the population variance
  - recognize that n, the sample size, can be determined from the values of the sample means
- use linear and quadratic functions to model relationships between two quantitative variables presented numerically and/or graphically
  - decide whether a linear or quadratic model or neither is appropriate for data presented in a table or graph

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- recognize an appropriate algebraic model given a table or graph (note: in the absence of appropriate technology, students will be expected to estimate the correct parameters for a linear or quadratic function presented in the form $y = ax + b$ or $y = ax^2 + bx + c$)
  - decide whether or not a particular set of data is appropriately modeled by a given function
- understand issues that arise when using data to explore the relationship between two variables, including correlation, e.g., recognizing whether the fit of an algebraic model is strong, weak, or nonexistent; confusing correlation and causation

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